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Formulario di Fisica Moderna

Decadimenti $M \to m_1 + m_2$; $\alpha =$ num. part.

$$p^{*2} = \frac{1}{4M^2}[M^4 + (m_1^2 - m_2^2)^2 - 2M^2(m_1^2 + m_2^2)]c$$

$$\mathcal{E}_1^* = \frac{M^2 + m_1^2 - m_2^2}{2M}c^2; \quad \mathcal{E}_2^* = \frac{M^2 - m_1^2 + m_2^2}{2M}c^2$$

$$\beta_M > \beta_\alpha^* = \frac{p^*}{\mathcal{E}_\alpha^*} \quad \text{(Emissione in avanti di } \alpha)$$

$$p_\% = 100 \cdot \frac{1}{2\beta\gamma p^*}\Delta E$$ (Percentuale di particelle -senza spin- generate ad energia nel range ΔE)

$$\mathcal{E}_\alpha = \gamma_{CM}(\mathcal{E}_\alpha^* + \beta_{CM} c p^* \cos\theta_\alpha^*)$$

$$p_{\alpha,x} = \gamma_{CM}\left(p^*\cos\theta_\alpha^* + \beta_{CM}\frac{\mathcal{E}_\alpha^*}{c}\right); \quad p_{\alpha,y} = p^* \sin\theta_\alpha^*$$

$$\tan\theta_\alpha = \frac{p_{\alpha,y}}{p_{\alpha,x}} = \frac{\sin\theta_\alpha^*}{\gamma\left(\cos\theta_\alpha^* + \frac{\beta}{\beta_\alpha^*}\right)}$$ (Angoli θ_α, θ_α^* sono rispetto a linea di volo)

$$\sin\theta_\alpha^{\max} = \frac{p^*}{m_\alpha \gamma_{CM}\beta_{CM}} = \frac{Mp^*}{m_\alpha p_M} \Rightarrow \cos\theta_\alpha^* = -\frac{\beta_\alpha^*}{\beta} \text{ (se } \theta_\alpha \text{ è max)}$$

-Masse uguali $M \to m + m$ (θ è angolo tra particelle)

$$|p_1^*| = |p_2^*| = p^* = \sqrt{\frac{M^2}{4} - m^2}; \quad \mathcal{E}_1^* = \mathcal{E}_2^* = \mathcal{E}^* = \frac{M}{2}; \quad \tan\theta(\theta^*) = \frac{A\sin\theta^*}{\sin^2\theta^* + B}$$

$$\beta_1^* = \beta_2^* = \beta^* = \frac{p^*}{\mathcal{E}^*} = \frac{2p^*}{M} = \sqrt{1 - \frac{4m^2}{M^2}}; \quad A = \frac{2}{\beta^*\beta\gamma}; \; B = \frac{1}{\beta^{*2}} - \frac{1}{\beta^2}$$

Urti (Elastici, bersaglio fermo) $\mathcal{E}_1 + m_2 \to \mathcal{E}_1' + \mathcal{E}_2'$

$$p^{*2} = \frac{m_2^2(\mathcal{E}_1^2 - m_1^2)}{m_1^2 + m_2^2 + 2m_2\mathcal{E}_1}; \quad \beta_{CM} = \frac{|\vec{p}_1|}{\mathcal{E}_1 + m_2}; \quad \sin\theta_{1,max} = \frac{p^*}{cm_1\beta\gamma} \; (m_1 > m_2)$$

$$\mathcal{E}_1' = \mathcal{E}_1 - \frac{p^{*2}}{m_2}(1 - \cos\theta^*); \quad \mathcal{E}_2' = (\mathcal{E}_1 + \mathcal{E}_2) - \mathcal{E}_1'; \quad \sin\theta_{2,max} = \frac{\pi}{2}$$

$$\mathcal{E}_1' = \frac{(\mathcal{E}_1 + m_2)(m_1^2 + m_2\mathcal{E}_1) + p^2\cos\theta\sqrt{m_2^2 - m_1^2\sin^2\theta}}{(\mathcal{E}_1 + m_2)^2 - p^2\cos^2\theta}$$

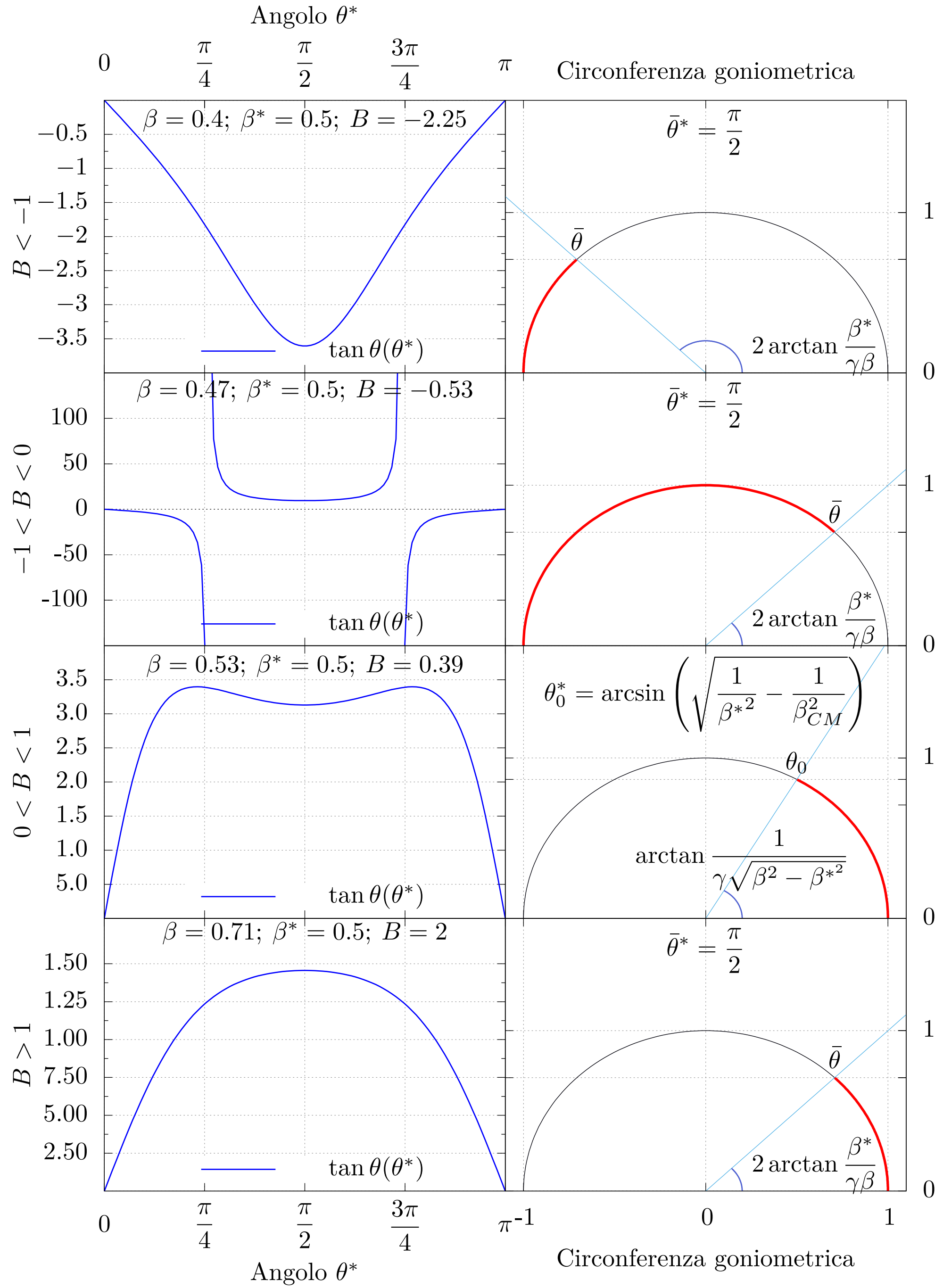
$$\mathcal{E}_2' = m_2\left(\frac{(\mathcal{E}_1 + m_2)^2 + p^2\cos^2\varphi}{(\mathcal{E}_1 + m_2)^2 - p^2\cos^2\varphi}\right)$$ (θ, ϕ angoli di particella 1 e 2 rispetto a direz. di volo)

-Collisione frontale (stesse masse) $m_1 + m_1 \to m_2' + m_2'$

$$\mathcal{E}_1^* = \mathcal{E}_2^* = \mathcal{E}_1'^* = \mathcal{E}_2'^* \quad (= m_2)$$ se si ha la produzione in soglia

$$\mathcal{E}_1 = \gamma(\mathcal{E}^* + \beta p^*); \quad p_1 = \gamma(\beta\mathcal{E}^* + p^*); \quad \beta_{CM} = \frac{p_{tot}}{\mathcal{E}_{tot}}$$

$$\mathcal{E}_2 = \gamma(\mathcal{E}^* - \beta p^*); \quad p_2 = \gamma(\beta\mathcal{E}^* - p^*)$$



Dinamica

$$x^\mu = (ct, x, y, z)$$

$$ds = \sqrt{dx_\mu dx^\nu} = \frac{c\,dt}{\gamma(v)}$$

$$\beta = \frac{v}{c};\ \ \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$u^\mu = \frac{dx^\mu}{ds} = \gamma(v)\left(1, \frac{\vec{v}}{c}\right)$$

$$w^\mu = \frac{du^\mu}{ds} = \left(\frac{\gamma^4}{c^3}\vec{v}\cdot\vec{a}, \frac{\gamma^2}{c^2}a^i + \frac{\gamma^4}{c^4}v^i\vec{v}\cdot\vec{a}\right)$$

$$p^\mu = mcu^\mu = \left(\frac{E}{c}, m\gamma(v)v^i\right)$$

$$\mathcal{E} = \sqrt{m^2c^4 + c^2|\vec{p}|^2} = m\gamma(v)c^2$$

$$\mathcal{F}^\mu = \frac{dp^\mu}{ds} = \left(\frac{\gamma}{c^2}\vec{F}\cdot\vec{v}, \frac{\gamma}{c}\vec{F}\right)$$

$$\frac{dp^\mu}{ds} = \frac{q}{c}F^{\mu\nu}u_\nu$$

$$u^\mu u_\mu = 1; \quad w_\mu u^\mu = 0$$

$$\begin{cases} \dfrac{d\vec{p}}{ds} = \dfrac{\gamma}{c}q(\vec{E} + \dfrac{\vec{v}}{c}\times\vec{B}) \\ \dfrac{d\mathcal{E}}{dt} = q\vec{E}\cdot\vec{v} \end{cases}$$

• Invarianti

$$p^\mu p_\mu = m^2c^2$$

$$\vec{E}\cdot\vec{B}; \quad E^2 - B^2$$

-Urti

$$p_1 + p_2 \to p_3 + p_4$$

$$W^2 = s = (p_1+p_2)^2 = (p_3+p_4)^2$$

$$t = (p_1-p_3)^2 = (p_4-p_2)^2$$

$$u = (p_1-p_4)^2 = (p_3-p_2)^2$$

• Formule generali

$$N(t) = N_0 \exp -t/\tau$$

$$\tau = 1/\lambda; \quad t_{\frac{1}{2}} = \ln(2\tau)$$

(τ = tempo di decadimento, misurato nel sdr in cui la particella è in **quiete**. λ = cost. di decad., $t_{1/2}$ = tempo di dimezzamento)

Relazioni utili

$$\beta = \frac{v}{c} = c\frac{p}{E} \Rightarrow \beta = \frac{p}{E};\ \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\Rightarrow \beta = \frac{\gamma^2 - 1}{\gamma^2}$$

$$\beta\gamma = A = \frac{p}{M} \Rightarrow \beta = \frac{A}{\sqrt{1+A^2}};\ \gamma = \sqrt{A^2+1}$$

$$\mathcal{E} = \sqrt{m^2c^4 + c^2|\vec{p}|^2}$$

Equazioni di Maxwell

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho \quad \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\vec{B}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial}{\partial t}\vec{E}$$

$$\vec{F} = q\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right)$$

$$\int_\Sigma \vec{E}\cdot d\vec{\Sigma} = 4\pi\int_\tau \rho d\tau$$

$$\oint_\gamma \vec{B}\cdot\vec{s} = \frac{1}{c}\left(4\pi\int_\Sigma \vec{J}\cdot d\vec{\Sigma} + \frac{\partial}{\partial t}\int_\Sigma \vec{E}\cdot d\vec{\Sigma}\right)$$

Composizione velocità

$$\begin{cases} v'_x = \dfrac{v_x - V}{1 - \frac{Vv_x}{c^2}} \\ v'_y = v_y\dfrac{\sqrt{1-\frac{V^2}{c^2}}}{1-\frac{Vv_x}{c^2}} \\ v'_z = v_z\dfrac{\sqrt{1-\frac{V^2}{c^2}}}{1-\frac{Vv_x}{c^2}} \end{cases}$$

Trasformazioni (boost lungo $\hat{x}$)

$$x = \gamma(x' + vt) \qquad x' = \gamma(x - vt)$$

$$t = \gamma\left(t' + \frac{v}{c^2}x'\right) \qquad t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$x' = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x; \quad x = \begin{bmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x'$$

$$\begin{cases} E'_x = E_x \\ E'_y = \gamma(E_y - \beta B_z) \\ E'_z = \gamma(E_z + \beta B_y) \end{cases} \qquad \begin{cases} B'_x = B_x \\ B'_y = \gamma(B_y + \beta E_z) \\ B'_z = \gamma(B_z - \beta E_y) \end{cases}$$

Se $\vec{E}\cdot\vec{B} = 0$ (campi $\perp$) $\to$ si può annullare il campo con modulo minore.
Se $E < B$, allora $E' = 0$, $B' = \sqrt{B^2 - E^2} = B/\gamma$, $\beta = E/B$
Se $B < E$, allora $E' = \sqrt{E^2 - B^2} = E/\gamma$, $B' = 0$, $\beta = B/E$.
(**Nota**: vanno trasformate anche le condizioni iniziali!)

Moto in campo $\vec{E}$ (Campo lungo $\hat{x}$)

$$\frac{d\vec{p}}{dt} = q\vec{E}; \quad \frac{d\mathcal{E}}{dt} = q(\vec{E}\cdot\vec{v}) \Rightarrow \Delta\mathcal{E} = qE\Delta x$$

(Vale anche in presenza di $\vec{B}$ costante)

$$\begin{cases} p_x(t) = qEt \\ p_y(t) = p_{0y} \\ p_z(t) = 0 \end{cases} \qquad v_x(t) = \frac{c^2(qEt)}{\mathcal{E}_0\left[1 + \left(\dfrac{cqEt}{\mathcal{E}_0}\right)^2\right]^{1/2}}$$

$$\begin{cases} x(t) = x_0 + \dfrac{1}{\alpha}(\sqrt{1+(\alpha ct)^2} - 1) \\ y(t) = y_0 + \dfrac{p_{0y}c}{qE}\operatorname{arcsinh}(\alpha ct) \\ z(t) = z_0 \end{cases} \quad \text{con } \alpha = \frac{qE}{\mathcal{E}_0}$$

(Misurate tutte nel sdr in cui si misurano le distanze!)

$$x(t) = \frac{\mathcal{E}_0}{E}\left(\cosh\left(\frac{qEy(t)}{p_{0y}c}\right) - 1\right)$$

Moto in campo $\vec{B}$

$$\omega = \frac{qcB}{\mathcal{E}} = \frac{qB}{m\gamma(v)c}$$

(Grandezze rispetto allo stesso sdr!)

$$R = \frac{|v_\perp|}{\omega} = \frac{|v_\perp|m\gamma(v)c}{qB} = \frac{c|p_\perp|}{qB}$$

(Nota: proiezioni su assi diversi da quello di boost non variano)